THE YEAR'S PROGRESS IN THE MATHEMATICAL WORK OF THE UNIVERSITY HIGH SCHOOL

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To save the teachers concerned in the pedagogical enterprise now going on in the University High School from the charge of vainglory a few preliminary remarks may be allowed to put this report in the right light.

In the first place, these teachers feel that most of the benefit that has thus far accrued from concert of effort has been personal. It has shown them to themselves to the very considerable gain in their professional efficiency. It has keyed them to a livelier sense of needs, and to a keener appreciation of remedies. It has added to their professional skill in the arts of diagnosis, of prognosis, and of prescription. As to these benefits, however, it seems most fitting, professionally, to let the work speak for itself.

In the next place, the experiment is still in its comparative incipiency. Not to mention any of the numerous unavoidable interruptions and interferences that have beset the way, two years is too short a period to work out in theory and to test out in practice a modern curriculum for high-school mathematics. Much good theory can be spun in two years, but the practical trying-out of theory, under a sufficiently extended and varied set of conditions to give to it breadth and depth of interest, proceeds more leisurely. It is then, too soon to pronounce even with high probability what the outcome will be. There have been along the way criticisms enough to conserve in the participants a due modesty. Some of these criticisms have been off-hand, and hasty. Some have been warranted; and not a few of the most telling have been due to the unavoidable circumstances; that the new has been compelled to patch up the best it could with the old, while matters are forming, and that the new has

always been judged exclusively by old standards. For reasons, then, that are readily appreciable by all, the natural inclination of the teachers that are directly concerned in this experiment is just now to devote their energies rather to sifting these criticisms and to a careful scrutiny of their pedagogic practices in the light of them, than to public professions of progress already made—until a stronger case than is yet possible can be made out.

But after all, what should most concern teachers that are striving for better things is not so much the question either of compromising professional dignity, or of convincing others that they are right, as of getting themselves right; and then of sharing with "whom it may concern" the benefits of their experience. Newton was perhaps overmodest in declining to publish to the world the results of his researches out of fear of being "persecuted with discussions" arising out of his theories. Smaller minds are usually less troubled with undue modesty. In the present instance, at any rate, the persons involved have no serious anticipations of being so "persecuted" with the necessity of defending their findings as to be led to blame their "own imprudence for parting with so substantial a blessing" as their quiet "to run after a shadow." These persons will readily be pardoned the mild presumption of the claim that, under the inspiration of this experiment, they are rendering a higher quality of professional service than they could render without it.

Moreover, the editors of *The School Review* have assured them, while asking them to state through its pages what the year has wrought mathematically, that some of its readers will con the record with interest. If the editors prove to have been correct in their assurances, it is not too much to expect that the attritions of reciprocal experiences with a common problem may in some persons heighten interest, if not stir enthusiasm, for better results from the teaching of secondary mathematics.

PREPARATION FOR HIGH-SCHOOL MATHEMATICS PROPER

The current pedagogical dictum that *the elementary school is not to fit for the high school; but to fit for life,* when rightly interpreted, is sound. But current practice in the elementary

school frequently misinterprets it. Most elementary teachers know that this thought was first uttered as a protest against the too common practice, in our highly mechanized school system, of paying too great attention, in the training of children, to the preparation needed for the next higher grade, or division, of the school, and too little attention to the individual pupil. In applying the foregoing dictum in their practice, nowadays, many teachers unduly stress the phrase *not to fit for the high school.* The misinterpretation referred to results from the failure of many to see that the high school is *a part of life.* In case of not an inconsiderable number of boys and girls, it becomes a very important part of life. The interests of these boys and girls are as much—I was about to say "just a little more"—entitled to consideration, as are those of the boys and girls who, from necessity, or choice, cut the high school out of their lives. The sin of ignoring in the high schools the special interests of the boys and girls who will go on to college is no greater than that of overlooking the particular needs of those in the seventh and eighth grades who will go into the high school. To do the latter, is to go far toward narrowing the conception of the elementary school to that of a nursery of a "hopeless mediocrity." Such a conception of the elementary school even those who are ignoring the interests of the intending high-school pupil would resent. The elementary school is and must be kept a school of the best nurture in the best things *for all pupils—including those who will go on to high school.*

The pupils of the first year of the University High School come from a large number and variety of elementary schools. One of the most important findings of the year originates partly in this fact and, in still larger part, in the misconception on the part of many seventh- and eighth-grade teachers, of the manifest function of the elementary school, touching high-school interests. The year's work has revealed most conspicuously the need of giving more attention than has been the custom to gathering up, systematizing, and evening up, with reference to immediate mathematical purposes, the attainment of pupils just entering high school, before taking up high-school mathematics

proper. Too often pupils enter from the best elementary schools almost entirely devoid of the language of arithmetical science. An "up-to-date" elementary teacher seems to fear it will reflect professional discredit upon her to teach the standard processes of fractions, without which nothing mathematical beyond the merest rudiments of arithmetic can be handled. Factoring in the elementary schools seems to be rapidly becoming a lost art. Everything arithmetical in the grades is running to simple, little oral exercises that can be talked of without a moment's thought, and for which little or no school arithmetic is needed. In a word, arithmetic seems to be rapidly degenerating into mere labial and lingual exercises. And even the language used in the arithmetic is the commonest vernacular, not the language of the science at all. This leaves matters in a pretty bad state for high-school mathematics.

Accordingly, hereafter in the University High School a distinct feature of the mathematical work of the first year is to be the devoting of the first three or four weeks to a critical survey, with chief emphasis on the scientific side, of the most fundamental parts of arithmetic. The topics to be thus rigorously examined and summarized are: (1) the fundamental operations applied to integers and to fractions (both common and decimal); (2) tests of divisibility; (3) factoring; (4) greatest common divisor and least common multiple, by the method of factors, and (5) the language and terminology appropriate to these topics.

It is thought that after three or four weeks of this explicit scientific rounding-out of these topics, the work of finishing the mastery of them will be reduced to proportions making it practicable to conclude the arithmetical treatment of each topic as a preliminary to the algebraic, or geometric, treatment of the analogous topic. This period of explicit study of formal arithmetic will close with a series of tests that will guarantee, in those who stand them successfully, a modicum of much-needed attainment and some homogeneity.

The most opportune time for such work is, of course, in the grades as the closing feature of topical treatments of arithmetic.

Much of the arithmetical confusion that now afflicts elementary-school pupils is due to the fact that this concluding step is not taken in a large proportion of our elementary schools. In fact, the only escape from this confusion lies in doing just this pedagogical service to the child. Textbooks sometimes aid and abet in this fault by offering to the learner nothing but an unsystematized mass of problem-material, with little or no organization with reference to scientific arithmetical content. Over-spiralized arithmetics have wrought great havoc of late years in the educative worth of arithmetic, by eviscerating it of its scientific quality. From the point of view of pedagogics, of culture, or of practical and economical efficiency, the argument for the need of supplying this important step in elementary arithmetic teaching is the strongest. Unless this step is taken at the proper psychological time, the educational point in arithmetic teaching is never quite made.

Unfortunately, the remedy for this deficiency can scarcely be expected until teachers qualify for a higher degree of practical proficiency in the three classical R's. Many elementary-school teachers seem not to appreciate the important truth that quite as much help to practical teaching is to come from an earnest study of the pedagogics of the old subjects of instruction as of the new. Here as elsewhere the chase seems to be ever after the new. In our determination to preserve in the elementary school the best of the old and the best of the new, we cannot afford to forget that what is best today in the old can be made better just as surely as can the best of the new. Instead of catering so much to those who "wish to specialize" by bespattering themselves from six weeks to six months with one of these new subjects, may we not hope soon to see the normal schools and colleges of education giving much more aid and encouragement than is now given to the psychologizing and professionalizing of the standard subjects, and to the training of "special teachers of elementary school mathematics," of "special teachers of history," of "special teachers of geography," etc., for grade work? Happily, the urgent need for the once-dreaded "departmental work" gener-

ally in the elementary school is growing daily more keenly felt and more popular among educational experts. The above-cited finding is only another concrete manifestation of the need. The dangers, which were once feared by elementary-school experts, and which they sought to avoid by eliminating departmental teaching, vanish as teachers become thoroughly well qualified professionally for their work. The spectacle of a curriculum split apart longitudinally here and there by shortsighted departmental teaching, but still held together vertically with some tenacity, is less deplorable than that of a curriculum cross-sectioned horizontally in eight places before reaching high school and with only such feeble vertical coherency in the several lamina, as can be given by the necessarily shallow hold of a single teacher charged with professional responsibility for a dozen different branches and a half hundred pupils. No "general pedagogic proficiency," if indeed there be such a thing, can prove equal to so stupendous a task. The problem is a specific one, and needs explicit attention, particularly in the mathematical work of elementary schools.

THE FIRST YEAR'S WORK CRITICIZED

The work of the first year in secondary mathematics proper is precisely the subject-matter of the manual published by The University of Chicago Press, and entitled *First-Year Mathematics for Secondary Schools,* by the writer, assisted by the instructors in mathematics in the University High School. This book, which is a sort of first-fruits of the University High School functioning as a laboratory for educational experimentation, is put into the pupil's hands as soon as he has completed the arithmetical preliminaries referred to above. He comes now to the mathematical work of the high school with considerable knowledge of arithmetical number and of mensurational truths. To break away from all this for a year or more, by injecting algebra as an isolated and self-contained subject, as is the custom, is unpedagogical and wasteful. The authors of this book, which is the first of a contemplated series covering the entire field of secondary mathematics, believe that the best use

that can be made of the first-year pupil's time is: (1) in generalizing and extending arithmetical notions; (2) in following up the notions of mensuration into their geometrical consequences; (3) in reconnoitering a broadly interesting and useful field of algebra, and (4) in treating, with sufficient completeness for high schools, a large part of what is most practical and most useful in elementary algebra. This means postponing the scientific and purely logical aspects of algebra to a later period. Such definitions as are needed are taught just where they are needed, and not didactically, but through a range and variety of illustrative uses.

The sources from which problems are drawn are arithmetic, mensuration, geometry, physics, and elementary mechanics. The treatment of these materials is given unity and balance by distributing them throughout the text wherever they render most effective algebraic service. The equation is made the starting-point and agency for developing the topics considered. In other words, the attempt of this manual is to bring together, to enrich and extend, and to begin setting in order the quantitative side of the pupil's experience. The book is essentially an extensive and varied body of mathematical ideas correlated around an algebraic core. The treatment begins with the informal methods of inductive arithmetic, passes to the uses of the equation and to its transformations, and by degrees assumes a deductive character.

Thus it is seen that the authors regard deductive logic as something to be taught explicitly, but gradually. Two years of experience in practical use in all the first-year classes of the University High School have furnished convincing arguments to parents, teachers, and administrative officers of the practicability of a correlation plan of teaching first-year mathematics, and all who have been immediately concerned in the plan are certain of its superiority, educationally, over any sort of sequence of separate mathematical subjects.

CRITICISMS

1. Too few concrete exercises, or exercises with arithmetical numbers, before requiring pupils to generalize out the algebraic formulation of the truth, or principle, to be taught.

A single problem in arithmetical numbers used merely *to illustrate* an algebraic truth, or process, is only another form of didactics. The generalizing act is extremely difficult to high-school pupils whose antecedent instruction has been of the common *direct* variety involving nothing on the learner's part but to assent and to remember. To say nothing of the unjustifiableness scientifically of generalizing from one, or two, or even half a dozen particular instances, the unpracticed pupil fails utterly to bring the act of generalizing about without a large number of instances. Indeed, the better skilled one becomes in generalizing the more indispensable does he feel the need of a larger number of particulars. He is bound to feel, not merely that there is a truth imbedded in the illustrative exercise, but also that it is a manifold, a general, truth as well. Teaching, or rather explaining, the several topics of algebra by means of one, or even of two, illustrative problems is precisely parallel to the antiquated science teaching, in which the teacher himself merely performed an illustrative experiment before the eyes of the class.

For example on page 6, Problem 1 reads: "A boy rides on his bicycle 8 mi. in one hour and 5 mi. the next hour; how far does he ride in the two hours?" Then comes problem 2. "If the boy rides *a* mi. the first, and 7 mi. the second hour, how far does he ride in the two hours?" It is clear that there should have been three or four problems like 1, given in rapid succession before passing to problem 2, that the pupil may feel the *generality of the way of doing all* such problems. *One* illustrative problem misses the point, because it drops out the essential element in reaching it. It only *exhibits,* without *substantiating,* the truth. It simply *faces* the mental machinery in the right direction. The substantiating is the essential thing. This criticism is much more telling against any other high-school text on algebra, yet in print, even the youngest claimant to attention that is not yet half a year old, than against the present book. But it is real, and justifiable here. Authors are very loath to admit any *real induction* into algebra.

2. The chapter on "The Uses of Inequalities" is too difficult, because too abstract. This introductory chapter was inserted

here as a background against which the equation could early be thrown into bold relief. Until the pupil feels that there are other working relations between numbers than the equational relation, he cannot do more than take on faith that the equational relation is a real and important relation in problem-work. It was the purpose of the authors to impress the learner as early as possible in his career that the relations of the equation, of inequalities, and of proportionality constitute the essence of algebra from the view-point of its practical worth for problem-solving. One can no more feel the worth of anyone of these relations without another one of them to compare and to contrast it with, than he could become conscious of redness, if there were no other color than red in the world. The end sought is therefore only partially attained, because the treatment was made too abstract, and hence, too difficult. This will be remedied, of course.

3. There is not enough of practical geometry in the text.

Practical problems of a constructional, or mensurational, character appeal to the first-year pupil unquestionably with greater drawing force than do any other problems of the text. There should be considerable constructive work, such as drawing perpendiculars, parallels, rectangles, triangles, etc., under specified conditions. Almost none at all is called for by the text in its present form. Such work, so far from consuming too much time from the algebra proper, as was feared, very materially accelerates progress, by the greater clearness and steadiness of thinking which it begets.

4. From chapter xv on the proportion of verbal problems to formal problems is too small.

There is now no question as to whether a sufficient quantity of formal algebra can be covered during the year on a correlation plan. More formal algebra is covered by this text in the first year than is attempted by other texts. The vague fear of timid teachers, that the introduction into the elementary algebra of cognate material from arithmetic and geometry must contract by just so much the algebraic ground covered, can be set aside as a purely subjective phenomenon. More ground is better covered,

and more pleasurably covered, under the vitalizing influence of such work than can be done by narrowing the ken of the learner to pure algebra.

The difficulty met on the more restricted plan is that formal problems do not impress the learner as being problems at all, and, as enforced work, they color the pupil's thinking so faintly as to make an inordinately great quantity of them necessary to fix any ideas at all. The mode of teaching algebra by "a large number of easy formal problems" is inane and wasteful, because the ideas such problems generate are so scanty, so feeble, and so uninteresting that, to fix any notions at all, topics must be covered, and recovered, then reviewed, and re-reviewed so repeatedly as to tire the learner out with the profitless "threshing of the old straw." Relatively fewer problems—problems of a verbal sort that have in them enough vitality to stir in the learner mental reactions of a nature both positive and friendly—are what is needed in the latter half of the book. The present text errs less than the standard texts in this regard; but its faults are grievous enough to call for drastic corrective treatment. In the fear of not covering enough formal matter, the text has slipped too abruptly into the common mistake of over-formalism. Without going with some advocates of reformation to the extent of excluding all formal exercises, it is clear to those who have taught the book, that a very much larger proportion of its problems should be of the verbal sort, that require as an organic part of their solution the formulation of the necessary equations in the symbols of algebra. This brings us to the next criticism.

5. After completing the text, students have too little ability to translate verbal into formal algebraic statements and *vice versa.*

The text contains a large number of the formulas of physics. After receiving numerous complaints from the teachers of physics that pupils who had been given in the mathematical classes much training in handling precisely the equations and formulas that they were not dealing satisfactorily with in the physics, the writer reported the matter to the mathematics

teachers. After a little investigation it was found that precisely the same students were dealing with these equations and formulas in both physics and mathematics. To guard against such complaints—which were not new—the classes were being given a rigorous review as third-year students in the mathematical class. The situation that now developed was that in the mathematical class the teacher, no matter how much he strove to complicate matters, could not even perplex the students with these formulas. He reported: "They simply ate them." Nevertheless, in the physics classes the students had *very serious* difficulty with the formulas. The science teachers continued to complain.

The upshot of the inquiry was that the difficulty in the physics was at last located. It was found to be *in arriving at the formulas from the verbal statements and metrical truths* out of which the pupils were required to derive them. Concentrated practice in the mathematical class on the translation of verbal to formal statements and *vice versa,* furnished the desired relief very shortly. For the benefit of others who have not yet found relief from the charge of science teachers that third-year students of high-school mathematics cannot solve the simplest algebraic difficulties that arise in the science work, it may be remarked that the sort of modification of current algebra teaching, that will relieve the charge by removing its cause, is to give much more training in handling verbal problems, by requiring the pupil continually to translate verbal into symbolic language, and the reverse. The remedy is easily applied and, incidentally, the pupil will also be thereby greatly benefited mathematically.

THE SECOND YEAR'S EXPERIMENT IN RETROSPECT

In the second year the emphasis of attention shifts from the algebra to the geometry as a unifying agency. The arithmetical and algebraic elements are not dropped here, but are merely subordinated to geometrical demands and interests. Measurement, calculation, and equation-solving are faithfully kept up. But just as throughout the first-year work geometrical matters were made to serve the purpose of illuminating, illustrating, and

reinforcing a central body of algebraic truth, so here, all antecedent mathematical knowledge is to play this subordinate rôle to a central line of geometrical thought. Geometry gives trend, organization, and unity to the whole. Building directly on the conceptual work of the preceding year, the work of the second year gradually assumes a purely deductive and demonstrative character. As to scope, it covers plane geometry, but since it treats plane geometry very largely from the view-point of the necessities of the mensuration of the surfaces, and the volumes of the standard geometrical solids, the truths of solid geometry are not rigorously excluded. In connection with similarity, the trigonometry of plane right triangles is to be given. Only a start in this direction was made last year. So much for the ultimate purposes of the second-year work, and for the extent to which they were actually pursued practically.

As an immediate means of attacking the practical problem of approaching this end, it was determined to try last year the European plan of carrying geometry and algebra along through the second year side by side. To keep to the beaten track as closely as possible, since the problem in hand is fully as much one of practical availability as of scientific soundness, the five periods of forty-five minutes each, per week, available for mathematics were to be divided approximately into three periods to geometry to two periods to algebra and arithmetic. This was thought to be a safe means (1) of deriving directly from experience some definite ideas on the practicability under American conditions of the plan of long and approved standing in German and French schools; (2) of facilitating the work of teachers on the plan of unified algebra and geometry by keeping both subject-matters fresh in their minds through their daily professional duties; and (3) of holding, for the pupils, the algebraic ground already made during the first-year, and perhaps of advancing a little, while laying the geometrical foundations. This plan, to which this retrospect exclusively refers, was followed with modifications by individual teachers throughout the year, with the single exception of one teacher, whose antipathy to the plan would not allow him to try it out. Later

this teacher did a little with the plan, but he still feels it to be impracticable and more undesirable than the common plan. A by-product worth mentioning here is that the attitude of mind of an experimenter in practical education outweighs all other factors in determining the issue.

The other teachers found that in the present state of American textbook literature two textbooks with two independent, parallel lines of work are both distasteful to students and unsatisfactory as to results. In the first place, students keenly felt and freely stated that they could see no reason for passing abruptly each week from one of the subjects to the other. They no more than began to feel they were getting a sort of hold on one of the subjects until they were taken away from it long enough to forget much of what they had got from it. This annoyance was somewhat reduced by an agreement among the teachers that each should change from one subject to the other at such places as the change seemed natural and helpful, conforming, by and large, to the division of the recitation time of three hours to two hours between geometry and algebra. As mentioned, this helped, but it rapidly became clear to all that what was most needed was not a standard American text on algebra, but rather a compilation of problems of a kind to connect the two subjects, problems based on geometrical principles and thinking, and solvable algebraically. A very considerable list of such problems was originated and collated. This list has since grown to more than nine-hundred exercises, in which the formulation of the equations required demands geometrical knowledge and power, while the solutions call for practice in algebraic equations and principles to and through quadratics. This list is classified with reference to geometrical concepts and uses. It is published by The University of Chicago Press. Without disturbing present curricula in the least, mathematical teachers might well use this list as a means of keeping the algebra fresh, through use, during the second-year work in geometry, by assigning problems from it as exercises, or as home-work, supplementing the regular geometrical work.

Obviously, more will be known of the full purport of the

second-year work after the third-year work is taken up. It may, however, be safely said that the second-year class has covered, and in a better way, as much geometry during the year as classes did formerly, and that the pupils have advanced considerably in their practical hold on algebra through putting it to continual use in their geometry. This plan will be continued.

The phases of geometry that stir most interested activity in high-school students are its constructive, mensurational, and metrical phases. An interest and a faith in the science of deductive reasoning are essential antecedent conditions to its profitable study. Through those phases of geometry these necessary conditions can be most economically reached. Most of the teachers the past year followed the Sanders' text in geometry, but experiments were made with a subject-matter formed by rearranging the propositions and truths of geometry into an order demanded by mensurational needs. This aroused in pupils a real interest in, and a growing regard for, the practical utilities of geometry. Pupils were generally willing to take up the deductive proof of a proposition that was seen to be needed to find the areas of the surfaces, or the volumes, or to construct the parts of the surfaces of the standard solids. A text in this tenor, which is in the nature of an educational experiment, is now being worked out by the mathematical faculty of the School of Education for the second-year classes. As to subject-matter it may be well characterized as *geometrized* mathematics, with emphasis on practical uses. It will be modeled somewhat closely on the following tentative plan.

THE POINT OF VIEW

The areas, volumes and other properties needed in measuring numerous surfaces and solids may be expressed in terms of the areas and properties of a few simple surfaces and solids that may be regarded as standard forms. The standard solids are the cube, the square, and the oblique block (called parallelopipeds), the prism, the pyramid, the cylinder, the cone, and the sphere. The standard surfaces are the surfaces that bound these solids.

Geometry has for its chief office to show about these standard figures the following five things:

1. What their areas and volumes are;

2. Why these areas and volumes are what they are found to be;

3. How, with compasses and ruler, to construct these standard figures;

4. Why these constructions are correct; and

5. How the treatment of more complicated figures is reduced to the treatment of these standard figures.

Examine a cube and name the faces that make up its surface.

Examine and name the forms of the faces that make up the surfaces of the square block; of the oblique block; of the prism; of the pyramid; of the cylinder; of the cone; of the hemisphere.

On the surfaces of these solids the following plane surfaces have been found: the rectangle, the square, the triangle, the parallelogram, other 4-, 5-, 6-sided, etc., figures, and the circle.

The areas of these figures will now be studied. We first take up the rectangle.

Here will follow the treatment, for the commensurable cases, of the deductive proofs of the customary propositions on the areas of plane surfaces.

Definitions are to be given where they are first needed and perhaps summarized at the ends of chapters.

OUTLINE OF CHAPTERS FOR PLANE GEOMETRY

I. An introductory chapter on constructions.

Enough constructive exercises, without theory, will here be given to enable pupils to make drawings of the figures needed in the propositions, with sufficient precision to make possible as an approach to the deductive proof, an approximate metrical proof of the principle to be established.

II. The Mensuration of Figures, using only commensurable cases.

This chapter is to be begun essentially as suggested above under the caption: The Point of View.

III. A chapter on Standard Geometrical Methods.

IV. A chapter on the Circle.

V. A chapter on Similarity, with the Beginnings of Trigonometry.

VI. A chapter on Polygons in general.

VII. A chapter on Regular Polygons, and the Mensuration of the Circle.

VIII. Perhaps a chapter on Maxima, Minima, Symmetry, and Loci.

The treatment of solid geometry to follow later, cannot be now sketched with greater detail than to state that it will follow the general plan of mensurational needs, very similar to that given above for the plane geometry.

THE GENERAL PLAN AND METHOD OF TEACHING

As already hinted, a distinct feature of the plan of presenting deductive geometrical proofs is the mode of approaching the propositions, and of putting meaning into them for the pupil before he is called on to prove them. It consists in general of five different steps, viz.:

1. The figure required by the demonstration is first sketched, in the rough, in a way to exhibit clearly the conditions under which the truth in question is to be established.

2. A careful drawing is next made on paper, or on the blackboard with ruler and compasses, under the specified conditions, and the appropriate parts of the figures that are drawn (protractor admitted) are then carefully measured.

3. Pupils are now required to make the best possible inferences as to the conclusions that follow from conforming to the imposed conditions.

4. A correct enunciation of the principle to be established is here made, and

5. A deductive proof is then given in standard form.

This method is followed, with modifications and abridgments, until the pupils get into the spirit and meaning of deductive argumentation. The scheme gets at many of the possible virtues of geometrical education that standard and customary presentations, either miss entirely, or stress so slightly, that they

leave no permanent impress upon the learner. Space can permit only the mere mention of some of them here.

1. Training the sense of number, and of dependence of form and of magnitudes upon imposed limitations, or conditions.

2. Training explicitly of the faculties of observation, of inference, and of judgment.

3. Training in ways of discovering new geometrical truths and in using deductive reasoning to establish, or to overthrow, suspected truths.

4. Secures clearness of conception and firmness of grasp of the meaning of theorems to be proved.

5. Impresses the novice with the inadequacy of pure metrical means and with the necessity of demonstrative methods.

The mode of conducting the class-work, in so far as there is a mode in a definitive sense, is a combination of the laboratory, the experimental, the Socratic, and the class-recitation modes. In general, the plan of administering the class-work is that of developing with the class the theory of the subject, and of assigning home-work which is of the nature of exemplifications, illustrations, and exercises to clarify and to impress the theory, and of following out fuller consequences and developments of the theory. The pupils' efforts are thus steadied by the clear grasp of sound theory, worked out in no haphazard fashion by himself, but under the guidance of one who is in full possession of it. His independence and his self-confidence are strengthened by his unguided, individual work, which is taken up always with a clear grasp of a fairly well-wrought-out theory. This plan has been proved to be best balanced, most economical and at the same time the most productive of interest and profit to the learner of any that has yet been tried.

PLANS FOR THE ENSUING YEAR

The present purpose is to follow the procedure just sketched both as to subject-matter and method of presentation with both the first- and second-year classes. This covers the ground of *the requirements* of all pupils of the University High School as to mathematics.

For the elective courses of the third and fourth years, consisting of advanced algebra, plane trigonometry, solid geometry, and college algebra, standard texts will be used, and the customary procedure followed, with the exception that plane trigonometry will be given just after advanced algebra, and before solid geometry. This order was followed last year with so much better results than usual that it will be continued.

The emphasis that has been put upon the utilities of mathematics in the first two years has developed interests in the pupils for the applications of mathematics, that have made it desirable to introduce three new courses. A course in plane surveying, following immediately the course in plane trigonometry, and in a very real sense, merely continuing the plane trigonometry, which is taught largely through the uses of the subject in surveying, has proved to be very much needed. It will be given to fourth-year classes this year. This course in plane surveying will be followed by an optional course in observational, metrical, and descriptive astronomy. Finally, a course in high-school arithmetic of a fourth-year grade of difficulty will also be given. This course will be open to students of both the high school and the College of Education. It will include a brisk review of the essentials of arithmetic with reference to fundamental principles, and will treat certain topics, such as compound interest, annuities, averages, graphing, the progressions and logarithms, etc., that are deemed too abstract for elementary arithmetic. College algebra and solid geometry are given primarily for students intending to enter certain technological schools that have very high entrance requirements in mathematics.